3.15
Parametric Equations and Motion

Parametric Curve, Parametric Equations

The graph of the ordered pairs (x, y) where

$$x = f(t), \qquad y = g(t)$$

are functions defined on an interval I of t-values is a **parametric curve.** The equations are **parametric equations** for the curve, the variable t is a **parameter**, and I is the **parameter interval.**

Graphing Parametric Equations (show on calc.)

Given: $x = t^2 - 2$ and $y = 3t$

Graph: $-3 \le t \le 1$ $\qquad$ $-2 \le t \le 3$ $\qquad$ $-3 \le t \le 3$

Eliminating the Parameter:

Solve the first equation for t, and then substitute that expression for t into the second equation.

Example: $x = 1 - 2t$, $y = 2 - t$ (if no parameter interval is specified then the parameter t can take on all values which produce real numbers for x and y.)

Solution: Solve the first equation for t.

$$x = 1 - 2t$$

$$2t = 1 - x$$

$$t = \frac{1}{2}(1 - x)$$

Now we substitute this expression for t into the second equation:

$$y = 2 - t$$

$$y = 2 - \frac{1}{2}(1 - x)$$

$$y = 0.5x + 1.5$$

Eliminate the parameter and identify the graph of the parametric curve.

$x = 2 \cos t$ $\quad$ $y = 2 \sin t$ $\quad$ $0 \le t \le 2\pi$

Remember that:

$$\begin{aligned}&\sin^2\theta+\cos^2\theta=1\\&\textit{and}\\&\sin\theta=y,\cos\theta=x\\&\textit{so,}\\&x^2+y^2=1\end{aligned}$$

this is the equation of the <u>unit</u> circle.

If we take $\begin{aligned}x^2&=(2\cos t)^2\\y^2&=(2\sin t)^2\end{aligned}$, then $\begin{aligned}x^2+y^2&=4\cos^2 t+4\sin^2 t\\&=4(\cos^2 t+\sin^2 t)\\&=4(1)\end{aligned}$

$x^2+y^2=4$ is the equation of a circle with radius 2, centered at the origin. Also see pg. 261; Ex. 4

Simulating projectile motion (pg. 262)

The path of an object is modeled by:

$$x=(v_0\cos\theta)t,\quad y=-16t^2+(v_0\sin\theta)t+y_0$$

Where v_0 is the initial velocity, θ is the angle with the horizontal, and y_0 is the initial height.

Hitting a baseball (Example 6 pg. 262)